Gauge-Yukawa Unification

and
The Top Quark Mass [†]

JISUKE KUBO [(1)], MYRIAM MONDRAGÓN [(2)] and
GEORGE ZOUPANOS [(3),*]

[(1)] *College of Liberal Arts, Kanazawa University, 920-11 Kanazawa, Japan*

[(2)] *Institut für Theoretische Physik, Philosophenweg 16*
D-69120 Heidelberg, Germany

[(3)] *Physics Department, National Technical University*
GR-157 80 Zografou, Athens, Greece

Abstract

The principles of finiteness and reduction of couplings can be applied to achieve Gauge-Yukawa Unification. It is found that the observed top-bottom hierarchy and the top quark mass naturally follow if there exists Gauge-Yukawa Unification which is a simple functional relation among the gauge coupling and the Yukawa couplings of the third generation in various susy unified gauge models. We briefly outline the basic idea of these principles and present the main results of the Gauge-Yukawa Unified models that have recently been studied in detail.

Why is the top quark so heavy, and who orders the top-bottom hierarchy? These are the questions to which we have recently addressed ourselves [1)−3)]. Obviously, these questions cannot be answered within the framework of the traditional GUT idea and new proposals are required going beyond GUTs. In a series of our resent studies [1)−3)], we have found that in a class of susy unified gauge models the top-bottom hierarchy as well as the top mass, consistent with the present experimental knowledge [4)], can be predicted, if the Yukawa couplings of the third generation are related in a certain way to the gauge couplings of the standard models at the unification scale–very similarly to the way the hierarchy of the gauge couplings follows in GUTs [5)]. This observation might indicate that Gauge-Yukawa-Unification (GYU) has a realistic meaning, as far as the Yukawa couplings of the third generation are concerned.

In these studies [1)−3)], we have considered the GYU which is based on the principles of reduction of couplings [6),7),2),3)] and also finiteness [8)−11),1)]. These principles, which are formulated in perturbation theory, are not explicit symmetry principles, although they might imply symmetries. The former principle is based on the existence of renormalization group invariant (RGI) relations among couplings which preserve perturbative renormalizability. Similarly, the latter one is based on the fact that it is possible to find RGI relations among couplings that keep finiteness in perturbation theory, even to all orders [11)]. Applying these principles, one can relate the gauge and Yukawa couplings without introducing necessarily a symmetry, thereby improving the predictive power of a model. In what follows, we briefly outline the basic tool of this GYU scheme and its application to various models.

A RGI relation among couplings can be expressed in an implicit form

$$\Phi(g_1, \cdots, g_N) = 0 , \tag{1}$$

which has to satisfy the partial differential equation (PDE) $\mu \, d\Phi/d\mu = \sum_{i=1}^{N} \beta_i \, \partial\Phi/\partial g_i = 0$, where β_i is the β-function of g_i. There exist $(N-1)$ independent Φ's, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations [6)],

$$\beta_g \frac{dg_i}{dg} = \beta_i , \quad i = 1, \cdots, N , \tag{2}$$

where g and β_g are the primary coupling and its β-function, and i does not include it. Using all the $(N-1)\,\Phi$'s to impose RGI relations, one can in principle express all the couplings in terms of a single coupling g. The complete reduction, which formally preserve perturbative renormalizability, can be achieved by demanding power series solution

$$g_i = \sum_{n=0}^{\infty} \kappa_i^{(n)} g^{2n+1} \ , \tag{3}$$

where the uniqueness of such a power series solution can be investigated at the one-loop level[6]. The completely reduced theory contains only one independent coupling with the corresponding β-function. In susy Yang-Mills theories with a simple gauge group, something more drastic can happen; the vanishing of the β-function to all orders in perturbation theory, if all the one-loop anomalous dimensions of the matter fields in the completely, uniquely reduced theory identically vanish [11].

This possibility of coupling unification is attractive, but it can be too restrictive and hence unrealistic. To overcome this problem, one may use fewer Φ's as RGI constraints. This is the idea of partial reduction [7,2),3)], and the power series solution (3) becomes in this case

$$g_i = \sum_{n=0}^{\infty} \kappa_i^{(n)} (g_a/g)\, g^{2n+1} \ , \ i = 1, \cdots, N' \ , \ a = N+1, \cdots, N \ . \tag{4}$$

The coefficient functions $\kappa_i^{(n)}$ are required to be unique power series in g_a/g so that the g_a's can be regarded as perturbations to the completely reduced system in which the g_a's identically vanish. In the following, we would like to consider three different GYU models.

A. Finite Unified Theory (FUT) based on $SU(5)$ [1]

This is a $N=1$ susy Yang-Mills theory based on $SU(5)$ [9] which contains one **24**, four pairs of $(\mathbf{5} + \overline{\mathbf{5}})$-Higgses and three $(\overline{\mathbf{5}} + \mathbf{10})$'s for three fermion generations. The unique power series solution [1], which looks realistic as a first approximation, corresponds to the Yukawa matrices without intergenerational mixing, and yields in the one-loop approximation

$$g_t^2 = g_c^2 = g_u^2 = \frac{8}{5}g^2 \ , \ g_b^2 = g_s^2 = g_d^2 = g_\tau^2 = g_\mu^2 = g_e^2 = \frac{6}{5}g^2 \ , \tag{5}$$

where g_i's stand for the Yukawa couplings. At first sight, this GYU seems to lead to unacceptable predictions of the fermion masses. But this is not the case, because each generation has an own pair of $(\overline{\mathbf{5}} + \mathbf{5})$-Higgses so that one may assume [10),1)] that after the diagonalization of the Higgs fields the effective theory is exactly MSSM, where the pair of its Higgs supermultiplets mainly stems from the $(\mathbf{5} + \overline{\mathbf{5}})$ which couples to the third fermion generation. (The Yukawa couplings of the first two generations can be regarded as free parameters.) The predictions of m_t and m_b for various m_{SUSY} are given in table 1.

m_{SUSY} [GeV]	$\alpha_3(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	m_b [GeV]	m_t [GeV]
200	0.123	53.7	2.25×10^{16}	5.2	184.0
500	0.118	54.2	1.45×10^{16}	5.1	184.4

Table 1. The predictions for $m_{\mathrm{SUSY}} = 200$ and 500 GeV for FUT.

B. Partially reduced Dimopoulos-Georgi-Sakai (DGS) Model [2)]

The field content is minimal. Neglecting the Cabibbo-Kobayashi-Maskawa mixing, there are six Yukawa and two Higgs couplings at the beginning. We then require GYU to occur among the Yukawa couplings of the third generation and the gauge coupling. We also require the theory to be completely asymptotically free. In the one-loop approximation, the GYU yields $g_{t,b}^2 = \sum_{m,n=1}^{\infty} \kappa_{t,b}^{(m,n)} \, h^m \, f^n \, g^2$. ($h$ and f are related to the Higgs couplings.) h is allowed to vary from 0 to $15/7$, while f may vary from 0 to a maximum which depends on h and vanishes at $h = 15/7$. As a result, we obtain [2)]

$$0.97 \, g^2 \lesssim g_t^2 \lesssim 1.37 \, g^2 \ , \ 0.57 \, g^2 \lesssim g_b^2 = g_\tau^2 \lesssim 0.97 \, g^2 \ . \tag{6}$$

In table 2, we give some representative predictions.

m_{SUSY} [GeV]	g_t^2/g^2	g_b^2/g^2	$\alpha_3(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	m_b [GeV]	m_t [GeV]
300	1.37	0.97	0.120	52.2	1.9×10^{16}	5.2	182.8
300	0.97	0.57	0.120	47.7	1.8×10^{16}	5.4	179.7
500	1.37	0.97	0.118	52.4	1.43×10^{16}	5.1	182.7
500	0.97	0.57	0.118	47.7	1.39×10^{16}	5.3	178.9

Table 2. The predictions of the partially reduced DGS model

4

C. Partially reduced susy Pati-Salam (PS) Model [3]

This is a model without covering GUT [12], that is, there is no gauge coupling uni-fication as it stands. The field content is [3]: three $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and three $(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ under $SU(4) \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ for three fermion generations, a set of $(\mathbf{4}, \mathbf{2}, \mathbf{1})$, $(\overline{\mathbf{4}}, \mathbf{2}, \mathbf{1})$ and two $(\mathbf{15}, \mathbf{1}, \mathbf{1})$ for Higgses that are responsible for the spontaneous symmetry breaking down to $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_Y$, and also a set of $(\mathbf{1}, \mathbf{2}, \mathbf{2})$, $(\mathbf{15}, \mathbf{2}, \mathbf{2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1})$. The singlet supermultiplet mixes with the right-handed neutrino supermultiplets at a high energy scale, while $(\mathbf{15}, \mathbf{2}, \mathbf{2})$ is introduced to realize the Georgi-Jarlskog type ansatz for the fermion mass matrix.

In one-loop order, we first obtain the unification of the gauge couplings,

$$g_4^2 \;=\; \frac{8}{9}\, g_{2\mathrm{L}}^2 \;,\; g_{2\mathrm{R}}^2 \;=\; \frac{4}{5}\, g_{2\mathrm{L}}^2 \;. \tag{7}$$

In the Yukawa sector, we find

$$2.8\, g_{2\mathrm{L}}^2 \lesssim g_t^2 \;=\; g_b^2 \;=\; g_\tau^2 \lesssim 3.5\, g_{2\mathrm{L}}^2 \;. \tag{8}$$

The typical predictions are presented in table 3.

m_{SUSY} [GeV]	$g_t^2/g_{2\mathrm{L}}^2$	$\alpha_3(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	m_b [GeV]	m_t [GeV]
500	2.8	0.129	61.2	0.16×10^{16}	5.4	196.8
500	3.4	0.132	62.1	0.17×10^{16}	5.4	198.3
1600	2.8	0.114	62.5	0.07×10^{16}	4.8	192.7
1600	3.4	0.112	63.4	0.06×10^{16}	4.7	193.3

Table 3. The predictions of the partially reduced susy Pati-Salam model

In all of the analyses above, we have used the RG technique and regarded the GYU relations (5)-(8) as the boundary conditions holding at the unification scale M_{GUT}. We have assumed that it is possible to arrange the susy mass parameters along with the soft breaking terms in such a way that the desired symmetry breaking pattern really occurs, all the superpartners are unobservable at present energies, there is no contradiction with proton decay, and so forth. To simplify our numerical analysis we have also assumed a

unique threshold m_{SUSY} for all the superpartners. Then we have examined numerically the evolution of the gauge and Yukawa couplings below M_{GUT} including the two-loop effects.

References

[1] D. Kapetanakis, M. Mondragón and G. Zoupanos, Zeit. f. Phys. **C60** (1993) 181; in Proc. of the 28th Recontres de Moriond on Electroweak Interactions and Unified Theories, March 20-27, 1993, p. 479; M. Mondragón and G. Zoupanos, Nucl. Phys. **B** (Proc. Suppl) 37**C** (1995) 98.

[2] J. Kubo, M. Mondragón and G. Zoupanos, Nucl. Phys. **B424** (1994) 291.

[3] J. Kubo, M. Mondragón, N.D. Tracas and G. Zoupanos, Phys. Lett. **B342** (1995) 155.

[4] LEP Electroweak Working Group, talk given by D. Schaile at the 27th Int. Conf. on HEP (Glasgow '94); F. Abe *et al.*, *Observation of Top Quark Production in $\overline{p}p$ Collisions*, FERMILAB-95/022-E; J. Konigsberg, Talk given at this meeting; S. Abachi *et al.*, *Observation of the Top Quark*, FERMILAB-Pub-95/028-E; B. Klima, Talk given at this meeting.

[5] H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32** (1974) 438; H. Georgi, H. Quinn, S. Weinberg, Phys. Rev. Lett. **33** (1974) 451.

[6] W. Zimmermann, Commun. Math. Phys. **97** (1985) 211; R. Oehme and W. Zimmermann, Commun. Math. Phys. **97** (1985) 569.

[7] J. Kubo, K. Sibold and W. Zimmermann, Nucl. Phys. **B259** (1985) 331.

[8] A.J. Parkes and P.C. West, Phys. Lett. **B138** (1984) 99; Nucl. Phys. **B256** (1985) 340; D.R.T. Jones and A.J. Parkes, Phys. Lett. **B160** (1985) 267; D.R.T. Jones and L. Mezinescu, Phys. Lett. **B136** (1984) 242; **B138** (1984) 293; A.J. Parkes, Phys. Lett. **B156** (1985) 73; I. Jack and D.R.T. Jones, Phys. Lett. **B333** (1994) 372.

[9] S. Hamidi, J. Patera and J.H. Schwarz, Phys. Lett. **B141** (1984) 349; X.D. Jiang and X.J. Zhou, Phys. Lett. **B197** (1987) 156; **B216** (1989) 160.

[10] S. Hamidi and J.H. Schwarz, Phys. Lett. **B147** (1984) 301; D.R.T. Jones and S. Raby, Phys. Lett. **B143** (1984) 137; J.E. Björkman, D.R.T. Jones and S. Raby, Nucl. Phys. **B259** (1985) 503; J. León et al, Phys. Lett. **B156** (1985) 66.

[11] A.V. Ermushev, D.I. Kazakov and O.V. Tarasov, Nucl. Phys. **B281** (1987) 72; D.I. Kazakov, Mod. Phys. Lett. **A2** (1987) 663; Phys. Lett. **B179** (1986) 352; C. Lucchesi, O. Piguet and K. Sibold, Helv. Phys. Acta. **61** (1988) 321.

[12] J.C. Pati and A. Salam, Phys. Rev. Lett. **31** (1973) 661.